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Cognitive Effects of Sleep Deficiency on Students' Mathematical Proficiency: A Review of the Literature

Guillermo Farfan
Florida State University

Mathematical proficiency relies on a host of cognitive processes that are shared with other mental functions, but three, in particular, are thought to play a central role: memory, attention, and executive functioning. This paper reviews some of the medical, psychological, and educational research literature on sleep deficiency and how it may affect one or more of these cognitive processes. It aims to provide a framework for future investigations into the mental processes of school-aged (K-12) and college-aged students that may be at risk from inadequate or insufficient sleep. Implications for mathematics teaching and learning are also briefly discussed.

Keywords: Sleep, memory, attention, executive function, mathematics achievement

Despite the essential role of sleep in the lifespan development of human beings, several studies have shown that children in the U.S. are not getting enough sleep, which has been associated with increases in stress levels, negative moods, and obesity (Millman, 2005; Short & Louca, 2015). It is estimated that sleep-related problems affect between 25% and 40% of U.S. youth (Meltzer & Mindell, 2006). These problems are not limited to the U.S., as similar patterns of inadequate sleep have been found in European and Asian countries (Gradisar et al., 2011). Although an increase in the use of electronic media and caffeine consumption has been cited as possible culprits in keeping some school-aged children awake longer (Calamaro et al., 2009), biological factors—such as changes associated with delayed evening onset of melatonin secretion—may be playing a role too (Owens, 2014; Owens et al., 2010).

In the sleep research literature, sleep deficiency is an umbrella term covering all phenomena related to inadequate or insufficient sleep. This includes sleep deprivation, sleep loss, sleeping at the wrong time of day, and sleep disorders that cause poor quality of sleep or disturbances in the sleep cycle (National Institutes of Health, 2011). Although less common, studies on the effects of sleep deficiency on academic performance are also found in the literature, primarily in adolescents and college students (Dewald et al., 2014; Kopasz et al., 2010; Meijer, 2008; Ming et al., 2011; Owens, 2014).

However, in these studies, the relationship between sleep and academic performance is often measured in broad terms, not independently of other measures. There is scant work on how poor or insufficient sleep may be affecting children's proficiency in specific subject areas, such as in mathematics. As an illustration, a quick search in one of the top journals in the field, Behavioral Sleep Medicine, returned only nine articles listed with the words "sleep," "school," and "mathematics," from 2007 to 2020. Of those, only two articles included measures to examine the relationship between sleep loss and primary students' academic performance in mathematics (Hiscock et al., 2018; Wolfson et al., 2007). A similar search for articles in another top journal, Journal of Sleep Research, returned five results of which only one publication involved using standardized measures of mathematics achievement to check for correlations with sleep deficiency (Erath et al., 2015).

One reason for the shortage of sleep articles focusing on mathematics education is that accurately measuring sleep deficiency and its effects on academic performance is challenging (Meijer, 2008; Keller et al., 2008). Simultaneously, understanding the impact of sleep deficiency on academic subject areas has taken on a new salience as schools across the country are starting the school day earlier to meet curricular demands, cutting significantly into the sleep cycle of children (Owens et al., 2010). The school day is generally structured by grade level to cover varying subject areas at separate times of the day, and each subject area often uses different cognitive resources in distinctive ways (Klein, 2003; Meltzer et al., 2018). Thus, it should be expected that whatever adverse impact sleep deficiency has on children's academic performance, it would not affect all subject areas equally.

This paper reviews some of the medical, psychological, and educational research literature on sleep deficiency in school-aged (K-12) and college-aged students. The review focuses on a selective number of articles whose results have a bearing on the question: what cognitive effects of sleep deficiency may affect students' proficiency and academic achievement in mathematics the most? Articles consulted for this review ($N = 58$) were retrieved from the following databases: Educational Resources Information Center (ERIC), Google Scholar, the Medical Literature Analysis and Retrieval System Online (MEDLINE), and PsycINFO. The original search was limited to the years 2007 to 2017, though an effort has been made to include more recent work. Keywords used to search and identify relevant literature were "sleep deficiency," "sleep loss," "sleep deprivation," AND "children," OR "adolescence," OR "young adults." Also, the following keywords were added for K-12 mathematics: "elementary school," "middle school," "high school," AND "mathematics."

The structure of the paper is the following: the first section briefly summarizes the importance of sleep for human welfare. The second section overviews relevant literature examining the relationship between sleep deficiency and three cognitive processes that play significant roles in mathematical proficiency: memory, attention, and executive functioning. Finally, the last section discusses some of the implications of this literature to the teaching and learning of mathematics, including suggestions on how to mitigate these effects.

The Importance of Sleep

Adequate sleep is vital to the physiological welfare of animals, including all vertebrates and even some invertebrates (Raizen, 2012). At least four stages of sleep have been identified in nature, according to the most recent classification (Schulz, 2008): three stages of non-rapid eye movement (NREM) and one stage of rapid eye movement (REM). Only humans and some mammals manifest all four stages and physiological sub-states of sleep, including those associated with slow-wave sleep (SWS), which is the last stage of NREM and most predominant in the early hours of sleep (Backhaus et al., 2007; Mader & Mader, 2016).

Sleep appears to have played many roles in the evolution of animals, such as metabolic homeostasis, defense and response to injury, and timed biological growth (Mader & Mader, 2016). Sleep also seems to have played an evolutionary role in both learning and memory consolidation, which is the stabilization of previously encoded information. Without consolidation, any positive or negative experience that an animal could learn from would be rapidly forgotten (Vorster & Born, 2015), which in turn lessens its chances for survival (Raizen, 2012).

In humans, consolidation of memories during sleep is dependent on the hippocampus (e.g., declarative memory) rather than memories that are independent of it (e.g., non-declarative memory; Carlson, 2013). Nonetheless, many studies have shown that both types of memories are boosted during sleep and that the prefrontal–hippocampal system is involved in both (Dotto, 1996;

Vorster & Born, 2015). At the same time, memory consolidation during sleep is not the same across the board but depends on several different factors, such as the kind of material or content to be learned and a person's age (Diekelmann et al., 2009).

Experts believe that children aged 5 to 10 years old should get between 9 to 10 hr sleep, while older children should get at least 8 hr (Biggs et al., 2011). It must be noted, however, that the recommended number of sleeping hours has trended downwards historically with little supporting evidence (Matricciani et al., 2012). Nonetheless, as will be discussed below, many adverse effects of poor or insufficient sleep are well documented (Astill et al., 2012; Gohar et al., 2009), including some of the neuro-mechanisms behind cognitive impairment after sleep loss (Halassa et al., 2009). Sleep deprivation, for instance, has been found to increase reaction times when accessing working memory for simple verbal and arithmetic tasks (Jiang et al., 2011), as well as altering one's mood and stress levels (Owens, 2014).

To study sleep, scientists rely heavily on electrophysiological instruments, such as electroencephalographic (EEG) and electrooculographic (EOG) recordings, as well as behavioral tasks and self-reported measures (Bryant & Gomez, 2015; Mader & Mader, 2016). Despite the number of tools at researchers' disposal, however, there are many unresolved questions regarding the effects of sleep deficiency on the academic performance of children, including in the subject area of mathematics, to which I now turn.

Cognitive Aspects of Sleep and Mathematical Proficiency

At its most basic level, mathematical thinking draws from many of the same cognitive resources that are available for thought processes in general (Phillips, 2014; Tall, 2013), such as symbolic and non-symbolic reasoning (Matejko & Ansari, 2017), visuospatial memory (Verdine et al., 2017), and language (Dehaene et al., 1999). Consequently, long-term development of mathematical proficiency is likely dependent on a subtle interplay of many cognitive and non-cognitive factors without relying on a fixed knowledge structure (Howes et al., 2019; Kilpatrick et al., 2001; Tall, 2013; cf. Schneider & McGrew, 2012).

Among the various cognitive processes underlying proficiency in mathematics (Schoenfeld, 1985), three, in particular, are thought to be closely associated with sleep: memory, attention, and executive functioning (Raghubar et al., 2010; Schmitt et al., 2017). The next section provides an overview of the literature on poor or insufficient sleep and how it may disrupt one or more of these three processes.

Memory

Memory, or the ability to encode, store, and retrieve information, is one area where substantial evidence on the effects of sleep deficiency has been collected (Rasch & Born, 2013; Swanson & Beebe-Frankenberger, 2004). It is known, for instance, that both SWS and REM sleep are involved in long-term memory consolidation, specifically declarative and non-declarative memories (Dotto, 1996; Maquet, 2001). For example, in mathematics, the memorization of number facts, terms, and formulas is a function of declarative memory (Andersson, 2010; Bayram, 2004). On the other hand, the computational aspects of an algorithm (e.g., long division) involve several mechanical steps that rely in part on non-declarative or procedural memory (Evans & Ullman, 2016; Van Putten et al., 2005). Since both types of memory seem susceptible to loss of SWS and REM sleep (Kopasz et al., 2010), it could be expected that reducing sleep hours will have some effect on children's memory consolidation and thus on their mathematical proficiency.

Studies combining electrophysiological and behavioral measures have shown that SWS particularly enhances declarative memories, such as cued recall, whereas REM sleep preferentially supports non-declarative memories, such as procedural and emotional aspects of memory (Maquet, 2001; Deak & Stickgold, 2010). Significantly, the strength of these associations appears to vary with age. For instance, an experimental study investigated the effects of SWS on declarative memories among 16 middle-aged (ages 48 to 55) and 14 young adults (ages 18 to 25) by observing participants' recall ability after two nights of sleep under laboratory conditions (Backhaus et al., 2007). Each subject in the study spent one night of short sleep (3.5 hr) and another night of late sleep (two sessions of 3.5 hr each) in counterbalanced order, with a cool-off period of one week between sessions. Using a word-pair cued recall task to test for declarative memory, the authors reported that memory retrieval in the early sleep condition among middle-aged adults was significantly worse than in young adults after controlling for other factors, suggesting that the former's declarative memories benefitted less from SWS overall (Backhaus et al., 2007). These results, which have been corroborated elsewhere (Mander et al., 2017; Rasch & Born, 2013; Spencer et al., 2007), indicate a likely change in the functional link between SWS and declarative memory over time. Presumably, as people age, there is less benefit from SWS because they encode less information ("synaptic downscaling;" Backhaus et al., 2007; Scullin, 2013), particularly if the information to be encoded is non-episodic in nature (Aly & Moscovitch, 2010; Diekelmann et al., 2009).

There is also evidence that younger children benefit from sleep differently than do older children and adults. In another study (Wilhelm et al., 2008), 15 children (ages 6 to 8, 9 females) and 15 young adults (ages 25 to 28, 13 females) were recruited to examine the dissociated effects of sleep on procedural and declarative memories after learning. The authors used a word-pair associate learning task and a 2D object location task to assess participants' declarative memories and a finger sequence tapping task to assess their procedural memories. For each participant, learning of tasks took place either right before sleep ("sleep" condition) or after awakening in the morning ("wake" condition), alternating the order in which participants experienced these conditions (Wilhelm et al., 2008). As expected, recall in the word-pair and 2D object tasks were better for both age groups in the sleep condition, but the same was not true on the procedural task: despite spending more time in both SWS and REM sleep, young children performed worse overall in finger tapping in the sleep condition than in the wake condition, which is the opposite to young adults (Wilhelm et al., 2008). The authors suggested that procedural encoding in young children may take place during waking hours rather than during REM sleep. These differential effects have been observed in other studies with young children (Astill et al., 2012; Fischer et al., 2007; Rasch & Born, 2013).

Together, these reports suggest that the relationship between long-term memory consolidation and SWS and REM sleep varies with age. Indeed, there is considerable evidence that such a relationship is at its strongest during adolescence and that adolescents are highly capable of compensating for short periods of severe sleep loss or deprivation (Astill et al., 2012; Gradisar et al., 2008; Wilhelm et al., 2012). Importantly, adolescence is also a critical period in children's mathematics education as many of the subjects taught during these years (e.g., algebra, geometry) serve as a foundation for future studies in college and beyond (Bayram, 2004; Bush & Karp, 2013; Welder, 2012). Moreover, as mentioned earlier, students make use of both declarative and non-declarative memories to master these subjects (Andersson, 2010; Evans & Ullman, 2016). Given that sleep deficiency curtails both SWS and REM sleep, it may have negative consequences on children's consolidation of these memories and indirectly interfere with their learning of mathematics.

Not only is sleep deficiency potentially damaging to long-term memory consolidation, but it may also have negative effects on working memory (W.M.), which is the ability to temporarily maintain and manipulate pertinent information (De Smedt et al., 2009; Unsworth & Redick, 2017). The importance of W.M. for mathematics proficiency is well documented. It is known, for instance, that W.M. plays an important role in computation and problem-solving skills (Rassmussen & Bisanz, 2005; Spybrook, 2008), and children with mathematics learning disabilities often display difficulties in their W.M. (De Smedt et al., 2009; Swanson & Beebe-Frankenberger, 2004); additionally, W.M. capacity is a strong predictor of later mathematics achievement (Friso-van den Bos et al., 2013; Passolunghi et al., 2007).

Evidence suggests that poor or insufficient sleep negatively affects W.M. (De Bruin et al., 2017; Kopasz et al., 2010), but it remains unclear what amount of sleep loss is detrimental. For example, there is evidence that adolescents tend to do worse in W.M. tasks when they report being repeatedly sleep-deprived. In a study by Gradisar et al. (2008), 143 adolescents, ages 13 to 18 (M = 14.9 years old, 81 females), were classified as sufficient sleepers (> 9 hr), borderline sleepers (8 to 9 hr), or insufficient sleepers (< 8 hr) based on their responses to a sleep survey. Using both a letter-number sequence and an operation span task to assess W.M., the authors found that adolescents in the insufficient sleep category performed worse, on average, on both W.M. tasks compared to borderline sleepers (Gradisar et al., 2008). This was most evident in the operation span task, which required participants to shift their attention from word list items to mathematical equations. The authors reported insufficient sleepers scored worse overall on this task than borderline sleepers and that the effect size was large (d = 0.92) even after controlling for general intelligence. However, there was no significant difference in performance between insufficient and sufficient sleepers in the operation span task, suggesting that frequently sleeping more than 8 or 9 hours may in some cases be as damaging to W.M. as sleeping less (Gradisar et al., 2008; see also Dewald-Kaufmann et al., 2013; Van Oostrom et al., 2018).

Conversely, there are instances where children can compensate for short periods of sleep loss while engaging their W.M. (Astill et al., 2012; Kopasz et al., 2010). A paper by Steenari et al. (2003) examined the relationships between quality and quantity of sleep and performance in auditory and visual W.M. tasks among 60 children, ages 6 to 13 (M = 9.9 years old, 31 females). To measure their sleep, children wore wrist-sized actigraphy devices for 72 consecutive hours. The W.M. tasks consisted of either auditory or visual items with three different cognitive load levels, with the number of items increasing or decreasing while keeping the other features constant (n-back task paradigm). Children performed these tasks on a computer on separate days under controlled conditions (Steenari et al., 2003). The authors found that impairment of W.M. was only noticeable in those with acute sleep loss during the highest cognitive load tasks, and that time spent asleep (i.e., all NREM and REM stages) was a better predictor of W.M. performance than total sleep duration (which includes time before falling asleep and shortly after waking up; Steenari et al., 2003). This is consistent with other studies that observed compensatory mechanisms among adolescents with chronic sleep loss during W.M. tasks, similar to those found in adults (Beebe et al., 2009; De Bruin et al., 2017; Kopasz et al., 2010).

In sum, poor or insufficient sleep often affects SWS and REM sleep, which in turn may interfere with the consolidation of declarative and non-declarative memories that are needed to learn mathematics. Moreover, while short-term compensation is possible, sustained sleep loss can also decrease W.M. performance, which could result in a failure to remember instructions, lack of coordination, and an inability to keep track of and complete mathematics learning activities (De Smedt et al., 2009; Swanson & Beebe-Frankenberger, 2004).

Attention

Another cognitive process that is important for mathematical proficiency is attention (Mason, 2003; Schoenfeld, 1985). Also known as vigilance, attention is the ability to select and retain relevant information for prolonged periods, and it can be further separated into three types: attention to relevant stimuli only (selective attention), attention to two or more tasks simultaneously (divided attention), and attention to one specific task without distraction (sustained attention; Belísio et al., 2016; De Bruin et al., 2017). To measure the effects of poor or insufficient sleep on attention, researchers often make use of psychomotor tasks that require quick responses to stimuli without getting distracted, selection and discrimination tasks, and cognitive and neuropsychological models (Belísio et al., 2016; Peng & Miller, 2016). These measures are frequently combined with psychometric scales to yield more accurate results (Jugovac & Cavallero, 2012).

In mathematics, attention regularly plays a critical role as it helps students focus, for example, on certain features that may be mathematically relevant for a task (e.g., vertices on a polygon) while disregarding others that are less likely to be so (e.g., color of lines in a figure). Moreover, it has been shown that attention correlates with numerosity and number processing (Anobile et al., 2013), that attention training can, in some cases, increase student mathematical performance (Zilaey et al., 2017), and that attention deficits often predict students' difficulty with solving complex mathematical tasks (Geary et al., 2007).

Evidence does exist that poor or insufficient sleep can have detrimental effects on attention. For instance, Jugovac and Cavallero (2012) examined the impact of one night of sleep deprivation on young adults' ability to achieve an alert state (alerting), select relevant information (orienting), and resolve conflicting stimuli (executive control). Thirty college students (ages 20 to 29 years old, 24 female) were assigned to one night of regular sleep (at home) and one night without sleep (in the laboratory), in alternating order (Jugovac & Cavallero, 2012). In addition to sleepiness and mood scales, participants completed a battery of attention psychomotor tasks, which involved identifying the left/right direction of arrows on a screen preceded by a cue (how these cues were presented to participants prompted their alerting, orienting, or executive control mentioned earlier; Jugovac & Cavallero, 2012).

Results showed average reaction times and accuracy among young adults were lower after one night of sleep deprivation than under normal sleep conditions (Jugovac & Cavallero, 2012). However, a closer look revealed participants' attention being affected selectively: the authors found no significant differences in alerting or orienting performance under either sleep condition, whereas executive control significantly decreased after one night without sleep (Jugovac & Cavallero, 2012). This suggests that, rather than causing a general drop in attention, losing sleep led to a selective impairment in participants' executive control only. Although other studies have shown comparable results (Choudhary et al., 2016; Short & Banks, 2014), considerable variation in the ability to attend to and resolve conflicting stimuli has also been reported, often attributed to individual differences or task-specific features (Eaves et al., 2020; Whitney et al., 2017).

Similar adverse effects on attention have been observed in adolescents while emulating more realistic conditions. In a study by Agostini et al. (2017), the effects of insufficient sleep on adolescents' sustained attention were examined using a combination of polysomnography, psychomotor tasks, and a sleepiness scale. Twelve high-school students, ages 15 to 17 (M = 16.1 years old, six female), were recruited for a simulated school week under laboratory conditions comprising of two nights with 10 hr of sleep, followed by five nights with 5 hr of sleep, and another two nights with 10 hr of sleep. To measure attentional deficits, the authors used the Psychomotor Vigilance Task (PVT), which consisted of pressing a button as soon as a red counter appeared on the screen at random intervals of 1 to 10 s. The authors found that attentional lapses increased immediately after sleepless nights, with the worst PVT performance happening during the morning hours (Agostini et al., 2017).

Interestingly, the last two nights with 10 hr of sleep—which were meant to resemble a "weekend" of sleep recovery—were insufficient to restore baseline performance, even though participants subjectively thought it did (Agostini et al., 2017; see also Lo et al., 2016). This could have important implications for those students whose mathematics classes start early in the day, as they may find performing mathematical tasks that require sustained attention more difficult depending on the time of day, although the evidence for it remains inconclusive (Sjosten-Bell, 2005; Wile & Shouppe, 2011).

Aside from these findings, however, a few systematic and meta-analytical reviews have found mixed results with regards to the effects of sleep deficiency on attention in children and young adults. For instance, Astill et al. (2012) evaluated 86 studies with a total of 35,936 children (ages 5 to 12) to identify associations between sleep and measures of cognitive performance. Contrary to what is usually reported in adults, they found no significant relationship between sleep duration and sustained attention as measured by psychomotor tasks (Astill et al., 2012). De Bruin et al. (2017) examined similar associations across 16 studies involving adolescents (ages 10 to 19) and found that complete lack of sleep (deprivation) does affect attention in general, but a shorter sleep schedule did not. Finally, Lowe et al. (2017) reviewed 61 studies that include 1,688 participants of various ages (M = 28.74 years old) and found a significant negative effect of shorter sleep on sustained attention but no significant effects on selective or divided attention.

Accordingly, it appears that poor or insufficient sleep could have negative effects on some forms of attention (e.g., sustained), but it is unclear how severe or to what extent outside laboratory conditions (Belísio et al., 2016; Jugovac & Cavallero, 2012). Nonetheless, if attentional deficits do occur, they may persist and not be easily recuperated (Agostini et al., 2017; Lo et al., 2016). Because proficiency in mathematics often requires being able to attend to mathematically relevant details (Booth et al., 2017; Kaminski & Sloutsky, 2013), attentional deficits caused by poor or insufficient sleep could result in lower mathematics achievement over the long run.

Executive Functioning

Executive functioning (E.F.) is often described as an amalgamation of cognitive resources that are involved in goal-directed behavior, inhibitory control, and cognitive flexibility (Schmitt et al., 2017; De Bruin et al., 2017). As such, E.F. builds on the mechanisms of both memory and attention, such as retrieving relevant information during a task or suppressing likely distractors (Cragg et al., 2017). More importantly, mathematical proficiency often depends on an interplay between domain-general skills that E.F. presumably supports and domain-specific knowledge or skills particular to mathematics. For instance, solving a complex arithmetic problem may require selecting an appropriate strategy, holding interim solutions in mind during computation, ignoring unwanted number facts during retrieval, and switching between different number representations (Gilmore & Cragg, 2018). Because mathematics draws on such a wide array of underlying skills, many experts think children and young adults must rely on E.F. to become proficient in mathematics (Best et al., 2011; Schmitt et al., 2017).

Yet, given E.F.'s dependence on lower-level mechanisms, it is considerably more difficult to disentangle and assess the effects of sleep deficiency on E.F. than is the case with either attention or memory (De Bruin et al., 2017; Nguyen et al., 2019). Nonetheless, a handful of studies have attempted to examine whether poor or insufficient sleep negatively impacts E.F. A study by Tucker et al. (2010), for example, investigated the effects of sleep deprivation on E.F. in 23 young adults (ages 22 to 38, 11 female). Participants were randomly assigned to either two nights of total sleep deprivation or a sleep-as-usual (control) condition and spent six days in the laboratory completing a battery of tasks, some of which were designed to disassociate elements of their E.F. (Tucker et al., 2010). Results showed that overall performance on tasks declined during total sleep deprivation, but the effects of sleep deficiency on E.F. were not uniform across the board: impairment was observed in a psychomotor and a W.M. (Sternberg) task, but neither the time taken to retrieve an item from W.M. nor the ability to suppress likely distractors—core elements of E.F.—appeared significantly damaged despite total sleep deprivation (Tucker et al., 2010). Likewise, Skurvydas et al. (2020) randomly assigned 30 young adult males (*M* = 20.2 years old) to either one night of complete sleep deprivation or a control group to assess cognitive and psychomotor effects of sleep loss. The authors reported that young adults in the sleep deprivation condition significantly underperformed on E.F. tasks compared to control, but there were no significant differences in motor skill or attention tasks (Skurvydas et al., 2020).

Impairment of E.F. due to sleep loss has also been observed among adolescents. Cohen-Zion et al. (2016), for instance, examined the effects of insufficient sleep on the cognitive performance of 45 adolescents (*M* = 16.9 years old, 18 female) after four nights of restricted sleep (6 to 6.5 hr) and four nights of extended sleep (10 to 10.5 hr) in counterbalanced order. Using a battery of tests that included a Go-No Go task, a Stroop test, and a psychomotor task (Catch game) to assess E.F., the authors found that adolescents' ability to process information was significantly lower under the restricted sleep condition than the extended sleep condition and that these changes in information processing were positively correlated with their performance in E.F., motor skill, and attention tasks (Cohen-Zion et al., 2016). These results suggest that adolescents suffering from chronic sleep loss may have fewer available cognitive resources for E.F., presumably due to deficits in the prefrontal cortex (Satterfield & Killgore, 2019). This may cause them to reduce the amount of effort they put in processing and solving tasks that require E.F. (Cohen-Zion et al., 2016; Engle-Friedman et al., 2003).

There is also interest in how sleep helps or hinders the E.F. of young children, particularly because E.F. deficits are often associated with developmental disorders such as attention deficit hyperactivity disorder (ADHD), disruptive behavior disorder (DBD), and autism spectrum disorders (ASD; Kenworthy et al., 2014; Schoemaker et al., 2012). As is the case with adolescents and young adults, some evidence exists that poor or insufficient sleep negatively impacts children's E.F. For example, a five-day study by Sadeh et al. (2003) examined the effects of sleep restriction among 77 fourth- and sixth-grade children, ages 9 to 12 (*M* = 10.6 years old, 38 females). On the third night, participants were randomly assigned to either a sleep extension (1 hr more) or a sleep restriction (1 hr less) condition for the remaining of the study (Sadeh et al., 2003). Results showed that children in the sleep extension condition performed significantly better on a digit span E.F. task (measuring memory retrieval) and a continuous performance E.F. task (measuring sustained attention and inhibitory control) compared to children in the sleep restriction condition. The authors further noted that both E.F. tasks are highly correlated with reading and mathematics achievement (Sadeh et al., 2003).

Similarly, Molfese et al. (2013) examined the effects of a 1-hr sleep restriction on the brain activity of six young male children (ages 6 to 8) across three tasks measuring attention, speech perception, and E.F. (Directional Stroop test). Participants wore a geodesic sensor net and were assessed on these tasks after one week of an ideal 9:00 pm to 7:00 am sleep schedule and one week of a 1-hr delayed sleep schedule (tasks were presented in counterbalanced order). Results showed that children had less brain activity and performed worse in the Directional Stroop test following restricted sleep than after the ideal sleep schedule, suggesting that the attention and inhibitory control elements of their E.F. did not benefit from a shorter sleep period (Molfese et al., 2013).

On the whole, these results suggest that sleep deficiency may affect certain elements of E.F. in school-aged and college-aged students; however, it is important to note that research on sleep and

E.F. is not without controversy (Pace-Schott et al., 2009). Astill et al. (2012) noted in their meta-analysis that most studies report sleep duration but not time spent asleep, and for those who do report the latter, no significant correlations with E.F. were found. Another meta-analysis (Lowe et al., 2017) found sleep restriction had a small but significant negative effect on E.F.; however, this effect was inconsistent across studies. Finally, De Bruin et al.'s (2017) systematic review found some studies showing decrements in performance on E.F. tasks after poor or insufficient sleep, whereas in other similar studies using different E.F. tasks, there was no effect.

Some of these inconsistencies in results may be attributed to issues of measurement. As pointed out earlier, given that E.F. is thought to depend on other, lower-level cognitive mechanisms, disentangling what aspects of E.F. are directly affected by sleep deficiency would be difficult to do even in the event of reliable and valid measures (Nguyen et al., 2019). There is also the possibility that failure to observe effects on E.F. after little or no sleep might be attributed to individuals' use of coping strategies to help mitigate the negative effects (Pace-Schott et al., 2009). Nevertheless, empirical evidence supports the idea of a connection between poor or insufficient sleep and deficits in E.F. If so, this relationship can have important implications for mathematics achievement, as many of the skills needed for mathematical proficiency—from retrieving number facts to attending to structure—are connected in one way or another to students' E.F. (Cragg et al., 2017; Espy et al., 2004; Gilmore & Cragg, 2018).

Discussion

Mathematical thinking relies on a host of cognitive processes that are shared with other mental functions (Tall, 2013; Schoenfeld, 1985). Of those, three have been recognized in this paper as important for becoming proficient in mathematics: memory, attention, and executive functioning. The studies reviewed here investigated the effects of sleep deficiency on one or more of these cognitive processes and provided some important, albeit at times conflicting, results.

The effects of poor or insufficient sleep on memory, in particular, appear to have the most consistent results. There is strong evidence that SWS and REM sleep are essential for long-term memory consolidation and that restricted sleep can interfere with this process (Astill et al., 2012; Owens et al., 2010; Wilhelm et al., 2012). Additionally, the strength of the relationship between sleep and memory consolidation appears to vary with age, with adolescents benefiting the most from sleep (Gradisar et al., 2008; Wilhelm et al., 2012). Moreover, the timing and stages of sleep, rather than the gross amount of sleep hours, are more important for memory consolidation (Astill et al., 2012; Gruber et al., 2014). Working memory also seems affected by sleep deficiency, as a decrease in performance on W.M. tasks was observed after restricted or lack of sleep (Gradisar et al., 2008; Kopasz et al., 2010; Steenari et al., 2003). Although children and young adults are capable of compensating somewhat for these negative effects, sustained sleep loss could result in disruption of both declarative and non-declarative memories, as well as failure to recall or keep track of relevant information when completing mathematical tasks (De Smedt et al., 2009; Swanson & Beebe-Frankenberger, 2004).

The effects of sleep deficiency on attention and E.F. are less apparent but nonetheless seem to exist. For example, there is evidence that attentional deficits do occur after lack of sleep (Jugovac & Cavallero, 2012; Short & Banks, 2014), though these deficits may vary depending on the individual or task (Eaves et al., 2020; Whitney et al., 2017). Furthermore, a time for sleep recovery—such as a weekend—may be insufficient to recoup some of these deficits (Agostini et al., 2017; Lo et al., 2016). How much of this is long-lasting or translates into the classroom is difficult to ascertain, though given that mathematical tasks often require high levels of sustained attention, any deficits caused by poor or insufficient sleep could be detrimental (Belísio et al., 2016; Booth et al., 2017; Geary et al., 2007).

Similarly, there is evidence that poor or insufficient sleep can affect E.F. and thus interfere with mathematical proficiency. Some studies with children and young adults have observed declines in psychomotor and W.M. skills (Tucker et al., 2010), processing speeds (Cohen-Zion et al., 2016), inhibitory control (Sadeh et al., 2003), and brain activity (Molfese et al., 2013) after restricted or lack of sleep that correlates with poorer performance in E.F. tasks (Skurvydas et al. (2020). However, open questions remain as to whether sleep deficiency affects executive functioning inconsistently (De Bruin et al., 2017; Lowe et al., 2017) or in some cases not at all (Pace-Schott et al., 2009) due to measurement issues, or if the associations between performance in E.F. tasks and mathematics achievement is due to individual components of E.F. or a single latent E.F. factor (Nguyen et al., 2019).

It must be noted that the negative implications of sleep deficiency in children and young adults are in no way limited to the three cognitive processes highlighted here. There is mounting evidence that students who do not receive adequate sleep have poor academic performance, not just in mathematics (Bryant & Gomez, 2015; Dewald et al., 2014; Keller et al., 2015; Ming et al., 2011). Additionally, sleep deficiency in school-aged children—particularly adolescents—is frequently associated with adverse behavioral and affective well-being effects, including fatigue, irritability, depression, and anxiety (Gruber, 2013; Owens, 2014; Wrzus & Wagner, 2014). In turn, these effects may interfere with academic performance and success by, for instance, decreasing motivation and taking away the cognitive improvements brought by positive moods (Grootenboer & Hemmings, 2007; Pourtois et al., 2017; Singh et al., 2010). There is also evidence that sleep deficiency may affect male and female students differently (Bos et al., 2009; Meijer, 2008; Short & Louca, 2015).

In short, the findings reviewed here suggest that school-aged and college-aged mathematics students have much to lose (and nothing to gain) from poor or insufficient sleep. The cognitive effects of sleep deficiency can range from disruption of long-term memory consolidation to impairment of W.M. and E.F., and these effects may start showing in as little as having one hour less of sleep (Gradisar et al., 2008; Molfese et al., 2013; Sadeh et al., 2003).

Implications for Mathematics Teaching and Learning

Mathematics is one of the most cognitively demanding subjects in school (Andersson, 2010; Cragg et al., 2017; Gilmore & Cragg, 2018; Passolunghi et al., 2007), and students who suffer from poor or insufficient sleep may already be at a disadvantage cognitively compared to their peers (Erath et al., 2015; Lo et al., 2016; Maski & Kothare, 2013). This could manifest in several ways, such as difficulty accessing prior knowledge while problem-solving, frequent mistakes due to inattention, or slowness in completing routine mathematical tasks. Thankfully, the literature provides some direction as to what can be done to ameliorate the effects of sleep deficiency on students' proficiency and achievement in mathematics.

The most obvious recommendation is to allow children and young adults the opportunity to sleep adequately at night (Biggs et al., 2011; Matricciani et al., 2012). This may be easier said than done, as there might be a mismatch between the onset of biological changes and fixed school demands—for example, adolescents' willingness to stay up late but having to wake up at the same early time of the day (Borlase et al., 2013; Keller et al., 2015; Owens, 2014). Nonetheless, sleep hygiene such as not drinking caffeinated drinks or eating late in the evening, setting up a time for winding down, turning on blue light filters on display devices, or turning off devices are all recommended steps to ensure good quality of sleep (Bryant & Gomez, 2015; Owens, 2014; Suni, 2020).

Additionally, it is important to avoid oversleeping on non-school days, particularly for adolescents and young adults. This may seem paradoxical, as it is commonly believed that sleeping for longer on weekends can compensate for sleep loss; however, evidence suggests that doing so may exacerbate problems with falling asleep on time (Bryant & Gomez, 2015; Taylor et al., 2008), and in any case, oversleeping for one or two days appears ineffective in compensating for deficits accumulated during the week (Agostini et al., 2017; Lo et al., 2016).

Mathematics teachers can also assist students with poor or insufficient sleep by delaying the introduction of cognitively demanding tasks at the beginning of a lesson or avoid putting students who are known to suffer from sleep deficiency in overly stressful situations (Dewald et al., 2014; Maasar et al., 2019). This will depend on reliably identifying such students early, and there is some evidence that preventive, school-based sleep interventions can work in that regard (Gruber et al., 2016). Finally, delaying school starting times may allow students to sleep longer (Borlase et al., 2013; Bryant & Gomez, 2015; Wolfson et al., 2007); however, some studies found no evidence that time-of-day has any effect on mathematics achievement (Davis, 1987; Sjosten-Bell, 2005). This may be because delaying school start on its own does not guarantee children are getting sufficient sleep, as much still depends on other medical and socio-economic factors (Keller et al., 2015). Conversely, individual differences in mathematical ability and coping mechanisms also play significant roles in mathematics achievement (Gilmore et al., 2017; Skaalvik, 2018). Thus, overcoming the negative effects of poor or insufficient sleep most likely depends on a collective effort rather than on a single solution (Borlase et al., 2013).

Conclusion

The time spent sleeping constitutes about one-third of one's life, and yet many children and young adults today are getting less sleep than before (Millman, 2005; Owens, 2014). Proficiency in mathematics relies on cognitive processes that are often susceptible to sleep loss, particularly memory, attention, and executive functioning. By highlighting how poor or insufficient sleep may interfere with these processes, the literature reviewed here should be useful not only to researchers but to parents, students, and other stakeholders who get to make important decisions regarding sleep and mathematics achievement.

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